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Study on the optimization design method of multi-type-pile composite foundation[1]

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Abstract. The optimization design of multi-type-pile composite foundation should not only taken the constraint condition of bearing capacity and settlement into account, but also the constraint stress condition of the main pile. Exclude the load born by the soil between piles from upper load, the 2/3 of residual load is loaded on the main pile. Large stiffness should be used for main pile to reduce the settlement. Flexible type must be used for auxiliary pile if the pile bottom on the hard soil layer, however, the rigid type is another better choice. When the bearing capacity of soil is greater than 50 % of the total load or it is not greater than 50 % but soil is hard, single pile of composite foundation can be used. When the bearing capacity of soil is greater less 50 % of the total load, the multi-type-pile of composite foundation must be used.

Key words. Composite foundation, optimization design, design variable, constraint condition, objective function..

1. Introduction

At present, the research on optimization design method of composite foundation is mainly for the single pile of composite foundation. The research methods are usually used for theoretical analysis combined with the tests [1]–[3]. The mechanism of pile-soil interaction is analyzed by Konagai [4] and Cheung [5]. The three-dimensional bearing characteristics of CFG-lime Piles is investigated by Zheng via FEM [6]. On the basis of previous studies of pile-soil interaction, self-balanced design method of composite foundation considering pile upward and downward penetration is analyzed by Liu [7]. Based on the natural foundation and single pile $p-s$ curve,

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the design of composite foundation according to the deformation coordination of pile and soil is analyzed and optimized by Yang using "how much to fill the lack" approach [8]. Based on the existing research results of single pile composite foundation, the optimization design method of multi-type-pile composite foundation is discussed by this paper, and the constraint condition of main pile, and the selection of pile type and design scheme of composite foundation are also introduced in this paper.

2. Optimization design of multi-type-pile composite foundation

Optimization design usually includes design variables, constraint conditions and objective functions. Design variable is also called independent variable, it's the solution objects of optimization design; Constraints is the control condition of optimization design, for composite foundation, it mainly refers to bearing capacity and settlement. Objective functions is the purpose of optimization design, usually is the project's total cost.

2.1. Design variable

The problem of variable first for optimize the design is necessary to analyze. If all variables are taken into account, the calculation difficulty will certainly increase. So we usually take the variable that has greater impact on results as design variable, and take other variables as known conditions given in advance. Design variable usually needs a given values range, in which it can be continuous or discrete. A reasonable range of design variable can improve the computational efficiency.

For the design of multi-type-pile composite foundation, pile length, pile diameter, pile spacing, replacement ratio and the thickness and stiffness of cushion layer all have a certain impact on the foundation's bearing capacity and settlement. The following will make simple analysis of these five variables.

(1) Pile length

The effect of pile length on the settlement and bearing capacity of composite foundation is most significant. If the length is short, the underlying stratum's calculated thickness increased, the settlement of composite foundation increases and the bearing capacity decreases; with the increase of pile length, the composite foundation settlement and bearing capacity characteristics are strengthened, but the project cost increased accordingly. Because of the problem of effective pile length, too long pile length for the improvement of composite foundation bearing capacity is not obvious. Therefore, the pile length in composite foundation design is particularly important.

The values range should follow the following principles when pile length is design variable: (a) If there is a hard soil layer exist in the soil, we take its depth as the upper and lower limits of pile length; (b) If the soil layer is soft soil, for flexible piles, the upper and lower limits of piles length should include the effective length;

The limit of rigid piles values range can be appropriately relaxed and optimized it by trial calculation

(2) Pile diameter

Pile diameter affects the pile top stress of composite foundation. For flexible piles and rigid piles, pile diameter has less effect on bearing capacity and deformation than pile length, while it has particularly significant impact on the bearing capacity for discrete material pile.

(3) Pile spacing

With the increase of pile spacing, the pile-soil stress ratio increases, the composite foundation settlement also increases but the composite foundation bearing capacity reduces. However, in practical engineering design, the pile spacing is usually $3 \sim 4$ times the pile diameter. Therefore, pile spacing can't be used as design variable.

(4) Replacement ratio

Replacement ratio is determined by the pile diameter and pile spacing. Taking replacement ratio as design variable can not only consider the influence of pile diameter, but also consider the influence of pile spacing. In "Technical code for ground treatment of buildings" (JGJ79-2012) [9], the bearing capacity and composite modulus of composite foundation are both calculated by the replacement ratio. Therefore replacement ratio should be used as a design variable in the design of composite foundation.

(5) The thickness and stiffness of cushion

Cushion is a key technology in the design of composite foundation. The thickness and stiffness of cushion has important influence on the pile-soil stress ratio, the pile-soil load share ratio, and the efficiency factor of pile-soil bearing capacity. Thus, the theoretical calculation method considering the thickness and stiffness of cushion for the composite foundation settlement and bearing capacity is not perfect. We often use the numerical calculation method and experiment method at present [10], [11]. In this paper, the cushion is not designed as a design variable. In practical engineering, the thickness of cushion is usually $100 \sim 300$ mm. In order to make the bearing capacity of soil between piles play, in the design of composite foundation, if the pile stiffness is large, the cushion thickness of great value, on the contrary, the cushion thickness of small value. Some studies [12] suggest that the cushion thickness can be determined according to the ratio of pile diameter and cushion thickness and the value is $0.45 \sim 0.5$. The cushion material usually uses crushed stone. According to the above analysis, the effect of pile diameter and pile spacing are been included in the replacement ratio, they are not considered as design variables. The cushion thickness is not considered as design variable too, as it has a common values range in practical engineering. Therefore, pile length and replacement ratio are chosen as the design variables in the optimization design of multi-type-pile composite foundation.

2.2. Constraint Condition

In the design of composite foundation, it is necessary to meet the requirements of bearing capacity, settlement and foundation stability. The stability problems in construction engineering usually isn't prominent, mainly is the problem of bearing capacity and settlement. In these two problems, the problem of settlement is more important, and many buildings are damaged after completion due to excessive settlement. Therefore, it's necessary to establish constraint condition of bearing capacity and settlement in the optimization design of composite foundation. The constraint conditions of multi-type-pile composite foundation include two constraint conditions at least, namely settlement and bearing capacity

$$s \leq [s] \ , \tag{1}$$

$$p \leq f_{\mathrm{spk}} \ , \tag{2}$$

where s and f_{spk} are the settlement and bearing capacity of the composite foundation respectively, can be calculated in accordance with the relevant provisions of the "Technical code for ground treatment of buildings" (JGJ79-2012) [9]. s is allowable settlement of composite foundation; p is foundation pressure.

The biggest difference between multi-type-pile and single pile composite foundation is that there is the problem of main and auxiliary piles in multi-type-pile composite foundation. Main pile is "main defense line" of composite foundation because it plays a major role in composite foundation and is very important to the bearing capacity and settlement of composite foundation. Auxiliary pile plays a role similar to the soil between main piles. On one hand it strengthen the soil between piles and increases the modulus of composite foundation, on the other hand, it shares the load of the upper structure, reduces the main pile top pressure, and increases the bearing capacity of composite foundation. The auxiliary pile can be regarded as "secondary defense line" in the multi-type-pile composite foundation. Auxiliary piles with different type, the function is also different. When the auxiliary pile is granular material pile, according to the calculation of bearing capacity and composite modulus of "Technical code for ground treatment of buildings" (JGJ79-2012) [9], the main function is to strengthen the soil between piles, therefore, the calculation of multi-type-pile composite foundation is similar to single pile composite foundation. When the auxiliary pile is binding material pile, it not only plays the role of strengthening the soil between piles, but also can share the pressure of main piles and share the upper load wit main piles. In order to ensure the "main defense line" function of main piles in composite foundation, the design of the main piles should relatively strong. The constraint condition of main pile is as follows

$$R_{\mathrm{a1}} \geq \frac{2}{3} \frac{A_{\mathrm{p1}} \left[p - f_{\mathrm{sk}}(1 - m_1 - m_2)\right]}{m_1} \, , \tag{3}$$

where R_{a1} is the single pile's bearing capacity characteristic value of the main piles; p is foundation pressure. Under normal circumstances, bearing capacity of the soil between piles has been fully played when the bearing capacity of composite foundation is reached. Deduct the load born by the soil between piles from the upper load, and the remaining load is born by the main and auxiliary piles together. Equation (3) has stipulated the main pile to bear the residual load of 2/3, the purpose is to make the main piles to bear more load and to design the main piles relatively strong.

2.3. Objective function and the design step

Select the total cost of piles as objective function, assuming the unit area cost of the main and auxiliary pile respectively is α_1 and α_2 per meter, the expression of the objective function is

$$f(x) = (\alpha_1 m_1 l_1 + \alpha_2 m_2 l_2)BL\,. \tag{4}$$

Considering the length and width of foundation is fixed and the unit area cost of the main and auxiliary pile per meter is constant. For convenience, take the foundation length and width of 1 unit, the main and auxiliary pile of the same price per meter, for 1 unit too, then the formula (4) is rewritten as

$$f(x) = m_1 l_1 + m_2 l_2\,. \tag{5}$$

Based on the above analysis, the optimization design of multi-type-pile composite foundation can selected the main pile's length l_1, replacement ratio m_1 and the auxiliary pile's length l_2, replacement ratio m_2 as design variables. Take the settlement, bearing capacity and main pile sharing load as constraint condition, take the total cost of piles as objective function. The design variable $X = [x_1, x_2, x_3, x_4]^T = [m_1, m_2, l_1, l_2]^T$, the objective function $\min f(x) = m_1 l_1 + m_2 l_2$, the constraint condition $s \le [s]$, $p \le f_{\mathrm{spk}}$, $R_{\mathrm{a1}} \ge \frac{2}{3}\frac{A_{\mathrm{p1}}[p - f_{\mathrm{sk}}(1 - m_1 - m_2)]}{m_1}$. The specific design steps are as follows: (1) Give the upper and lower limit values of the main pile's length l_1, replacement ratio m_1 and the auxiliary pile's length l_2, replacement ratio m_2. At the same time give the values of other parameters. (2) Take the bearing capacity and the stress of main pile as constraint conditions, take the total cost of the piles as objective function to optimize the design, to solve the optimal length of long and short pile and replacement ratio. (3) Take the optimal length of long and short pile and the replacement ratio as constraint conditions to check the composite foundation settlement, end the calculation if the check passed. (4) If the check is not passed, the optimal length of long and short pile and the replacement ratio which calculated in step 2 are eliminated from the upper and lower limit values of the pile length and replacement ratio in step 1. (5) Repeat step $1 - 4$ until the result up to the requirements.

3. Discussion on related issues in design

3.1. The selection of pile type

When the single pile composite foundation is used, rigid pile is preferred. The stiffness of pile is more favorable for controlling the settlement of composite foundation. When a certain thickness of hard soil layer exists in the foundation, this layer can be used as the pile bearing stratum. When the strata is deep soft soil, the settlement of composite foundation can be satisfied by increasing the pile length and reducing the settlement of underlying stratum.

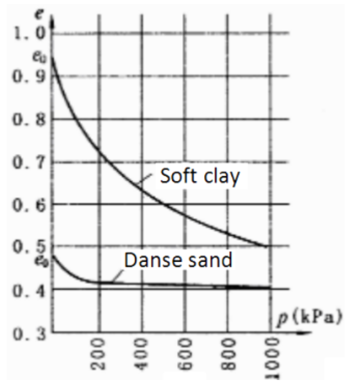

Fig. 1. $e - p$ curve of natural soil

When the multi-type-pile composite foundation is used, the main and auxiliary pile can be designed as the two defense lines. Main pile as "main defense line", share more load; auxiliary pile as "secondary defense line", share the top pressure of main piles. The "main defense line" should adopt the pile with larger stiffness to reduce the composite foundation settlement. The main piles often bear more loads and the pile top pressure is big. Under the action of pile top pressure, the pile top cushion layer is compacted gradually, as shown in Fig. 1. The compression amount of cushion layer is very small when the pile top pressure reaches a certain value, and thought it can't be compressed further. With the load continuous to increase, the composite foundation settlement mainly depends on the deformation of main pile. The larger stiffness of pile can reduce the composite foundation settlement.

We usually take half of the critical load of single pile to define the single pile bearing capacity characteristic value, that is to say the pile still has a certain bearing capacity even if the pile top load exceeds the single pile bearing capacity characteristic value. To take advantage of pile rigidity in this part, the auxiliary pile that as "secondary defense line" can use rigid pile with smaller rigidity or more weaker flexible pile relative to main pile. Since the pile top displacement is the sum of the pile downward penetration and the pile body compression, auxiliary pile can use flexible pile when the pile bottom soil layer is hard and can use rigid pile when this soil layer is soft. So, even the main pile reaches bearing capacity, because of its large rigidity, the deformation of composite foundation can be controlled. The deformation of the auxiliary pile and the soil can adjust the stress of the main pile, and the auxiliary pile also share part load of the main pile.

3.2. The selection of design scheme

For a practical treatment scheme, selecting single pile or multi-type-pile composite foundation is an important question. So, we can optimize the design of the single pile and multi-type-pile composite foundation separately in actual design, and then compare the schemes to choose a better one. When the bearing load of soil is greater than 50 % of the total load or it is not greater than 50 % but there is a certain thickness good soil layer that can be used as bearing stratum, we can consider the use of single pile composite foundation for design. But when the foundation is deep soft soil with high settlement requirements, multi-type-pile composite foundation should be adopt. The main pile can control the settlement and the auxiliary pile can ensure the bearing capacity of composite foundation. When the bearing load of soil is greater less 50 % of the total load, we give priority to use the multi-type-pile composite foundation. In actual design, we should consider geological conditions for comprehensive design. Generally, the bearing capacity of the soil between piles is fully play when the composite foundation reaches the bearing capacity characteristic value. The efficiency factor of bearing capacity of the soil between piles is 1. The contribution of the soil between piles to the composite foundation bearing capacity can be calculated by the following formula

$$\omega = (1 - m)\,\frac{f_{\mathrm{ak}}}{f_{\mathrm{spk}}}\,, \tag{6}$$

where ω is the contribution of the soil between piles to the composite foundation bearing capacity; f_{ak} is the bearing capacity characteristic value of natural soil; f_{spk} is the design value of composite foundation.

4. Example

The project is a 12-storey commercial-residential building on soft soil foundation, using raft foundation with 30.84 m in length and 14.7 m in width. The properties index of foundation soil is shown in Table 1. After the treatment, the bearing capacity of foundation is required to reach 212 kPa, the allowable settlement reaches 8 mm. The foundation buried depth is 3.35 m and the foundation pressure is 163.6 kPa.

According to the geological data, the soil between piles is muddy clay and its bearing capacity is 70 MPa. The foundation bearing capacity is required to reach 212 kPa after the treatment. The contribution of the soil between piles to the composite foundation bearing capacity can be calculated by the put the value into formula (6) to calculate

$$\omega = (1 - m)\,\frac{f_{\mathrm{ak}}}{f_{\mathrm{spk}}} = (1 - m)\,\frac{70}{212} < 50\%\,. \tag{7}$$

Table 1. Physical and mechanical properties index of soil

Soil layer number	Soil name	Average thick-ness (m)	Water content (%)	Natural density (kN/m^3)	Compression modulus (MPa)	Characteristic value of bearing ca-pacity of foundation (kPa)	Characteristic value of side friction (kPa)
1	miscellaneous pond mud	2.0					
2	silty clay	1.5	30.4	19.2	4.43	120	16
3-1	muddy clay	4.2	42.1	18.4	2.48	70	8
3-2	muddy silty clay	5.1	37.1	18.6	3.11	70	8
3-3	muddy silty clay	11.5	42.5	17.8	2.65	70	10
3-4	muddy silty clay	11.0	38.3	18.0	2.79	80	12
3-5	shell soil	2.2	44.8		2.81	80	15
6-2	clay (round gravel)	3.0			20.0	90 (300)	44
7	strong and medium -weathered rock					3000 (medium -weathered)	50

According to the content of the selection of design scheme in section 3.2, we should adopt the scheme of multi-type-pile composite foundation for this design. According to the content of the selection of pile type in section 3.1, the long pile should use rigid pile in the design of multi-type-pile composite foundation, and the short pile can be selected to use rigid or flexible pile according to the actual geological conditions. In this design, reinforced concrete pile is used as the long pile, the concrete compression modulus is 30000 MPa and the pile diameter is 0.5 m; the short pile adopts the cement-soil mixing pile, the pile compression modulus is 60 MPa and the pile diameter is 0.6 m. The short pile compression modulus is 60 MPa, the soil modulus takes the average modulus value of the 3-1 and the 3-2 soil layer, is 2.8 MPa. The pile-soil modulus ratio of short pile is 21. According to literature [13]–[18], the short pile effective length is 12 $d = 7.2$ m. The 3-2 muddy silty clay layer is selected as the bearing stratum of short piles, combined with the effective length of short piles, the upper and lower limit value of short piles calculation length is 7 m − 9 m. The 6-2 clay (round gravel) layer is selected as bearing stratum of long piles, the upper and lower limit value of long piles calculation length is 34 m − 37 m and the upper and lower limit value of the replacement ratio of long and short piles is 0.01 − 0.02. By the long pile diameter $D_1 = 0.5\,\mathrm{m}$, can obtain the long pile perimeter is $u_{\mathrm{p}1} = 1.57\,\mathrm{m}$ and the cross-sectional area $A_{\mathrm{p}1} = 0.196\,\mathrm{m}^2$. By the short pile diameter $D_2 = 0.6\,\mathrm{m}$, can obtain the short pile perimeter is $u_{\mathrm{p}2} = 1.88\,\mathrm{m}$ and the cross-sectional area $A_{\mathrm{p}2} = 0.283\,\mathrm{m}^2$. Assumption the length of the long and short pile respectively is l_1 and l_2, the single pile bearing capacity of the long and short pile can be obtained respectively

$$\begin{array}{c} R_{\mathrm{a}1} = 1.57\,[9.3\times 8 + 11.5\times 10 + 11\times 12 + 2.2\times 15 + (l_1 - 34)\times 44] + \\ +300\times 0.196 = 69 l_1 - 1734 \end{array}, \qquad (8)$$

$$R_{\mathrm{a}2} = 1.88\times 8\times l_2 + 70\times 0.283 = 15 l_2 + 20\,. \qquad (9)$$

The bearing capacity of composite foundation is

$$\begin{array}{c} f_{\mathrm{spk}} = m_1 \frac{\lambda_1 R_{\mathrm{a}1}}{A_{\mathrm{p}1}} + m_2 \frac{\lambda_2 R_{\mathrm{a}2}}{A_{\mathrm{p}2}} + \beta\,(1 - m_1 - m_2)\, f_{\mathrm{sk}} = \\ 282 m_1 l_1 + 37 m_2 l_2 - 7147 m_1 - 21 m_2 + 70 \end{array}. \qquad (10)$$

Therefore, the bearing capacity constraint condition is

$$282 m_1 l_1 + 37 m_2 l_2 - 7147 m_1 - 21 m_2 + 70 \geq 212\,. \qquad (11)$$

Put these conditions into formula (3), can obtain the constraint condition of the main pile

$$69 m_1 l_1 - 1743 m_1 - 9 m_2 - 19 \geq 0\,. \qquad (12)$$

Write the calculation program based on the Microsoft visual c + + as shown below:

```
#include <stdio.h>
void main()
{int m1,m2,l1,l2;
```

```
int m11,m22,l11,l22;
double z,zmin=1000000;
for(m1=1;m1<=20;m1++) {
    for(m2=1;m2<=20;m2++) {
        for(l1=340;l1<=370;l1++) {
            for(l2=70;l2<=90;l2++) {
                if((282*(m1/100.0)*(l1/10.0)+37*(m2/100.0)*(l2/10.0)···
                -7147*(m1/100.0)-21*(m2/100.0)-142>0) && ···
                (69*(m1/100.0)*(l1/10.0)-1743*(m1/100.0)-9*(m2/100.0)-19>0)) {
                    z=(m1/100.0)*(l1/10.0)+(m2/100.0)*(l2/10.0);
                    if(z<zmin) {
                        zmin=z;m11=m1;m22=m2;l11=l1;l22=l2;
                    }
                }
            }
        }
    }
    printf("m1*l1+m2*l 2minimum:%f\n",zmin);
    printf("now m1:%f\n",m11/100.0);
    printf("now l1:%f\n",l11/10.0);
    printf("now m2:%f\n",m22/100.0);
    printf("now l2:%f\n",l22/10.0);
}
```

Using above procedure can obtain: $m_1l_1 + m_2l_2 = 1.788$, $m_1 = 0.04$, $l_1 = 37$, $m_2 = 0.04$, $l_2 = 7.7$. According to the calculation method of settlement of "Technical code for ground treatment of buildings" (JGJ79-2012) [9], put the pile length and replacement ratio obtained by optimized design into it, obtain the composite foundation final settlement is 2.7 mm, meet the requirements of allowable settlement. So the optimal design of $m_1 = 0.04$, $l_1 = 37$, $m_2 = 0.04$, $l_2 = 7.7$. This design is the result of the material strength of long pile is 30000 MPa and the material strength of short pile is 60 MPa. You can also adjust the material strength of pile in actual design and the replacement ratio obtained from optimization design will be adjusted accordingly.

5. Conclusion

1. In order to ensure the "main defense line" function of main pile, the main pile should be designed for much higher stiffness than the auxiliary pile. The optimization design of composite foundation is studied in this paper in order to satisfy the requirement of bearing capacity, settlement and stress condition of main pile respectively. When excluding the load between the soil and piles from the upper load, 2/3 residual load of main pile must be satisfied.

2. The main and auxiliary piles can be designed as two defense lines for composite foundation. More load must be borne by the main pile, and the pressure of

main pile should be borne by auxiliary pile. The large stiffness main pile should be used to reduce the settlement of the composite foundation. The rigid type auxiliary pile must be used if the soil layer under the pile bottom is hard.

3. The soil layers can be used as bearing stratum if the bearing load of soil is greater than half of the total load or the soil layer is hard enough. The single pile can be used for composite foundation, but multi-type-pile should be used for the soft soil foundation with high settlement. The settlement can be reduced using the main pile, and the bearing capacity of composite foundation can be ensured by auxiliary pile. The multi-type-pile must be used for composite foundation if the bearing load of soil is less than half of the total load.

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